Mathematics in economics: some remarks

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Keywords *Economics theory, Mathematics, Economics*

Abstract *Attempts to throw some light on the sensible use of mathematics in economic theory. Argues that mathematics is a valuable and useful tool which economists should and must apply as long as its use is economically sensible. The dangers of going beyond the "frontier" of what is economically sensible occur when economists depart from the actual (empirical) subject matter because of the applied mathematical instruments, when the underlying value judgements are not, or only insufficiently, taken into consideration, when the recording and measurement of empirical magnitudes as an economic problem is underestimated or is even subordinate under the requirements of the formal language, and when the process of mathematization is considered as a substitute for the process of* Verstehen. *Concludes that although mathematical reasoning is one way of logical deduction, which secures a style of logical consistency in reasoning, it is a fallacy to believe that mathematical reasoning alone can secure logical, consistent reasoning. Mathematization for the sake of mathematization is useless.*

Introduction

Mathematics is a valuable and useful tool economists should and must apply as long as its use is economically sensible. This paper aims to throw some light on the limits of the sensible use of mathematics in economic theory. The danger of going beyond these limits appears when:

- economists depart from the actual (empirical) subject matter because of the applied mathematical instruments;
- the underlying value judgements are not or only insufficiently taken into consideration;
- the recording and measurement of empirical magnitudes as economic problem is underestimated or is even subordinate under the requirements of the formal language; and
- the process of mathematization is considered as a substitute for the process of "*Verstehen*".

The point of view which we argue from is that of an applied economist, and it is pragmatic as well.

Some general remarks

The use of mathematical methods in economics has increased greatly since the 1950s, whereas they were scarcely used at all during the whole of the nineteenth century (Fisher, 1941; Mirowski, 1989; 1991). Nowadays, the use of mathematics is often advanced dogmatically unquestioned (e.g. Debreu, 1986,

Journal of Economic Studies,
Vol. 27 No. 4/5, 2000, pp. 260-270.

p. 1261). For a large proportion of the economic profession involved in contemporary economic theory, mathematics has become the fundamental tool, best exemplified by economic textbooks such as *General Competitive Analysis* (Arrow and Hahn, 1971) and *Fundamental Methods of Mathematical Economics* (Chiang, 1984) or the *Handbook of Mathematical Economics* (Arrow and Intriligator, 1989). But mathematics has not only served as a tool for economics; economic thinking has repeatedly adopted new fields of mathematical thinking (Weintraub, 1998). From a mathematical point of view, however, economic theory has had no, or perhaps only a marginal, influence on mathematics.

As economists using mathematical methods, we think it is a necessary, and even inevitable, aspect of sound scientific practice to reflect on the meaning and practical relevance of mathematical methods in economics. Experience has taught us that sometimes (or even often) we are concerned with economic problems that could be better tackled by other methods (see, for example, Chick, 1998, p. 1860). But in our reservations regarding the use of mathematics in economics we would not want to go as far as McCloskey when he says, "economics is in love with the wrong mathematics, the pure rather than the applied" (McCloskey, 1994, p. 145), or as Rosenberg (1983, p. 311) who critically describes modern economics as "a branch of mathematics" (see more generally, Rosenberg (1979, pp. 526-7)). We would rather agree with the claim that the mathematization of economics has "gone a bit far" (Baumol, 1991, p. 2).

In trying to assess to what extent the use of mathematics makes sense and at which point it does not, we are willing to admit even at this point that we cannot be conclusive due to the fact that the evaluation of scientific method poses questions that are not definitely decidable. Different criteria for answering the question of the usefulness of mathematical methods concern, for example:

- Its contribution to scientific progress represented by indicators like: greater clarity of the arguments, reverting to the results of predecessors, ways of improving the model (Backhouse, 1998, p. 1851-2).
- Its economic efficiency, even extended to a cost-benefit analysis where the benefits are the scientific results and the costs are the expenditures of the scientific process. Indicators for the benefits are logical rigor and precision of arguments (e.g. Debreu, 1986, p. 1266-77), the costs are represented by all expenditures which were necessary to obtain the scientific results including the time and material expenditures for acquiring the mathematical skills. In recent years such costs have been lowered by the use of modern information technologies but imply the danger that economists uncritically use all the latest possibilities and "the computer revolution has, I believe, induced economists to carry reliance of mathematics and econometrics beyond the point of vanishing returns . . ." (Friedman, 1991, p. 36). At the same time Friedman emphasizes that those types of works which actually would generate

new knowledge collecting novel and original data, composing them to concepts and interpreting these concepts can not be replaced.

- As a further criterion for the evaluation of the use of mathematics in economics we consider its function as a tool for communication for economists to inform each other about their work, with special importance attached to communicating clearly the chain of reasoning which mathematics provides (Krugman, 1998, p. 1835).

This list is far from exhaustive and all criteria are highly interrelated and, finally, concentrate only on different aspects of the same problem. From an applied mathematician's point of view we choose the criterion of "realism" on all stages of scientific activity. By choosing this criterion we think/believe that especially both the criteria of scientific progress and of economic efficiency are to a great extent fulfilled.

Our discomfort with the use of mathematics

Because we do not agree with the uncritical use of mathematics within economics, we think it is necessary first to sketch the main aspects of our idea of a mathematical style of thinking. Mathematical reasoning encompasses:

- axiomatization;
- consistency in reasoning;
- making explicit what is hidden implicitly in the premisses;
- the importance of the chain of reasoning (proof procedure) as such;
- the convergence of necessary and sufficient conditions; and
- the greatest generality of the arguments.

Mathematical reasoning is logical deduction – this is a fact for a mathematician (at least for mathematicians in line of Hilbert, Peano and Russell where transformation rules indicate how to derive logical truths from logical truth). Mathematics as an axiomatic system can be considered a set of uninterpreted symbols and transformation rules which indicate how to generate new strings of symbols from given symbols. The symbols and strings of symbols can only be interpreted if a meaning is assigned to the symbols. Therefore, any statement in the form of symbol language is necessarily true. Even Albert Einstein remarked that, insofar as mathematical statements refer to reality, they are not certain, "and so far as they are certain, they do not refer to reality" (Einstein, 1974, p. 119-20, authors' translation). We conclude that mathematics as such is incapable of saying anything about reality. Also one has to agree with Drechsler's (1999, p. 6) and Dore's (1988, p. 457) objections to the use of mathematics, when mathematization – as the instrument that it is – is taken as a "safety foundation," "guaranteed truth" or a "truth content" or is taken as a proof of authority.

Let us start with some examples to underline our reservations concerning the careless use of mathematics in economics. In economics, it is usually accepted that the behavior of economic agents is determined by individual choices (methodological individualism) which in turn are dependent on the individual's wishes and preferences. We are faced with severe measurement problems if we try to represent the individual's choices and the underlying individual's preferences. To quantify variables like utility or individual or social welfare – and thereby deal with them in a sensible mathematical way – is a difficult process and often hard to do. If, for example, the term "welfare" is measured empirically, the concept of the consumer's surplus is often used. The concept of the consumer's surplus – if represented graphically or by mathematical symbols – gives a fairly good and intuitive idea what welfare could really mean. These forms of representation seem to give the impression of supposed exactitude and sufficient measurability. The terms "consumer's surplus" and "welfare" are equally difficult to measure for an empirically working economist and there is unanimity among empirically working economists that consumer's surplus is not to used as a measure of welfare although the reasons for that are controversially discussed (Slesnick, 1998, pp. 2108, 2110-13). This is not the case within the microeconomic and welfare theory where the concept is ubiquitous because of its formal-mathematical character, its intuitive plausibility and its alleged exactitude and measurability. The concept of the consumer's surplus is one of many cases which illustrates that mathematization is at best a necessary but not at all a sufficient condition in resolving economic problems.

To put the unchallenged law of the downward shaped demand curve (see, for example, Hutchison, 1977, p. 15; McCloskey, 1985, pp. 57-62) on empirical grounds, we are faced with similar obstacles. For the sake of simplicity, we assume that the demand curve is derived by the individuals' revealed preferences, that is to say without use of intensity of needs or on indifference curves). Samuelson derives the "general law of demand" or the "fundamental theorem of consumer theory" from a few assumptions: consumers prefer "more" to "less", choose a certain bundle of goods in each budget period and behave consistently (Samuelson, 1947, pp. 111, 116, 143-51, 172, 221; 1948, pp. 247-48, 251). The scientific progress concerning the empirical operationability is to derive preferences out of the revealed individual behavior. In addition, the income effect is measurable within the revealed preference theory. Samuelson succeeded in deriving the law of demand from some few logical requirements without using utility. This is certainly a gain in logical plausibility. But this "gain" has to be tempered by some critical objections – the income elasticity of demand of each individual has to be known (without this assumption no empirical prediction is possible). Furthermore, the axiomatization of the revealed preference analysis has reached a point where the premisses and conclusions are so close to one another that the truth content of one proposition is sufficient to derive the truth content of the other and vice versa (Houthakker, 1961, pp. 706-8).

Until now nothing was said about the fact that the preferences leading to choices are to a great extent expressions of social, cultural and anthropological phenomena which "precede the individuals". This is completely ignored by neoclassical and especially mathematical economics although the problem has been known since the Historical School and has been re-appreciated by critical economists again and again (for a survey, see Benton, 1990, pp. 77-8), let alone the contribution of sociology. Because the concept of preferences is not really traced back to its causes – and economic theory only considers preferences by their consistency conditions of completeness, reflexitivity, transitivity and even differentiability – empirical propositions about the stability of preferences are hardly possible and prediction is very problematic.

In spite of these critical aspects, the use of mathematics doubtless has advantages. Stignum (1990, pp. 15-16) mentions precision, verification, standardization of language, the discovery of new applications, and comparability. Debreu considers the developments in mathematics such as the requirements of axiomatization, generalization, weaker assumptions and stronger conclusions as an advatage for the use of mathematics in economics which can be found in the expansion of the two-consumer, two-person world to the general equilibrium (Debreu, 1991, p. 4). In general we agree with this enumeration in theory but, in empirical economics, the consequences are paramount. To clarify this point, we choose the example of generality, the achievement of which, to a large degree, is a fundamental aim of mathematics. Two things follow:

(1) Increasing generality of mathematical concepts leads to more understandable concepts (concepts more adequate to the object) and in this sense to scientific progress – here there is no problem at all.

(2) Increasing generality of mathematical concepts increases the complexity of the analysis as a consequence:

- For *mathematics*, scientific progress can be here ascertained provided that weakened assumptions lead to maintenance of results. This has to be seen behind the mathematical maxim that for a given result we have to reduce the gap between necessary and sufficient conditions. The ideal is to find conditions which are both necessary and sufficient.
- For *economics*, such logical progress (at the cost of an increasing complexity of analysis) has to be justified by an increase in the empirical relevance of the results. To be sure, analysis is also important in economic theory, but – as economists – we have to be aware of the fact that the generality of assumptions does not necessarily accompany empirical relevance in an economic context. In this case the question of a useful application of mathematics within economics arises. The use of mathematics in social sciences has too often tempted economists to generalize their assumptions with regard to the beauty of the theory. Beauty in the sense of mathematical generality with the consequence that the underlying actual problem is not at all represented. Within the optimization

> assumptions, e.g. when the condition of differentiability is replaced by the condition of convexity, the condition of continuousness is still maintained; although the use of discrete models is more adequate. If, on the other hand, economic relevance of results increases, then increasing complexity and generality are justified.

An analogous problem occurs in the foundation of the concept of probability. von Mises argued against Kolmogoroff's law of iterated logarithm rejecting it as a result of probability theory and reducing it to measure theory because of the non-verifiability of a zero set introduced during the proof (von Mises, 1964, p. 80).

In the case of the critically viewed application of mathematics to economics our argument – although coming from applied mathematics in the economic field – is of course derived from a normative point of view because calling for empirical evidence is nothing but an arbitrary claim but, we think, a reasonable one. Claiming these normative aspects is in contrast to positions insisting on a value-free judgement. Referring to an exact formulation of assumptions and conclusions through mathematization "economic analysis was sometimes brought closer to its ideology-free ideal" (Debreu, 1986, p. 1266). We argue that a value-free style of economic thinking can hardly be achieved, at least in an empirical and practical context, and that the translation of economic arguments into mathematical symbols cannot take away normative aspects from the arguments, much worse, it can even hide them. The archetypal problem of economics – the use of scarce resources as a problem of optimization – is the result of the (normative) decision. Optimization of scarce resources is not a mathematical project, it is the result of a normative view of economics, and mathematics just help to clarify the problems with this normative programme.

Such fundamental decisions concerning the usefulness of a normative programe are the representation of uncertainty through the concept of probability, the use of curves and functions as continuous and corresponding assumptions. A lot of mathematical efforts have been undertaken to conceptualize the category of uncertainty for use within the economic optimization paradigm. The basis of these efforts is an ideological one, a normative position, which says that probability is an adequate concept to modelize uncertainty. When using probability in real sciences one has to say what is expressed by probability. Neoclassical economic theory uses probability to choose between alternative decisions. This concept of probability is based on a betting approach influenced by data of experience. The influence of experience is modelized by Bayes statistics. What neoclassics does is substitute the utility concept with the concept of expected utility. The introduction of the expected utility implies the departure from an ordinal scaled utility concept to a quasi-cardinal scaled one. The fact that we are confronted with an normative rather than an objective process can be illustrated by the existence of at least one different fundamental concept of probability. Keynes as a theorist with a logical interpretation of probability does not admit that uncertainty can adequately be described by probability. For Keynes, probability is a logic connection between a sentence and a body of knowledge and measures the degree to which the sentence can be derived logically from the body of knowledge; it is only occasionally possible to measure the probability of a sentence (Keynes, 1973). This discussion about uncertainty has two consequences for the use of mathematics in economics:

(1) the use of mathematics forces acceptance mathematical premises (e.g. restriction of ordinality caused by linear instead of monotone transformation); and

(2) whether a mathematical concept can be used to formalize empirical phenomena adequately is a case to case decision, at least for the applied economist.

Measurement as a fundamental economic problem

A core problem of the mathematization of economics is the measurement of economic variables. The process of measurement itself (including recording) transforms the field of objects into abstract concepts which are expressed verbally (in a natural language) or by symbols (the latter is the case in mathematics). This process is very important for scientific reasoning because a lot of fundamental decisions and causes of misunderstandings can be found in this process. In using a certain method of measurement the scientist decides which elements or features of the object field are considered important and which are not. This decision is not neutral or objective because each decision about what has to be measured is inevitably affected by human judgement. Considering measurement in this way – as a decision process and basis of modeling within social sciences – entails a theoretical and philosophical task. Mathematics doesn't need to fill variables with empirical content, but economics must, because economics is a social and empirical science. The theoretical and philosophical task of measuring economic variables within (mathematical) economic modeling, deciding what is important and what is not, leads to the question of what is evaluated as relevant and irrelevant. We repeat: the mathematization of economics does not relieve economists of the task of ascribing an empirical content to the variables of their model.

In the field of applied mathematics (econometrics, operations research, etc.) it is necessary to specify kind, size and other attributes of variables and how to measure them (if at all possible), before introducing mathematics into economics. That means that measurement is not mathematics; it is rather the basis of modeling and a first step for the use of mathematics in (applied) economics. Decision about relevance can only be made on the basis of pre-judgments. These pre-judgments themselves are subject to variations and adjustments. Both the decisions about relevance and the pre-judgments are subject to a dialectical process, a process of *Verstehen*. This process is the result of considerations, pre-pre-judgements, previous decisions about relevance, their analysis and the experience of the surrounding world. Therefore, we think that there is no objective criterion for the appropriateness of decisions about relevance.

After transforming the object into a symbol field (through the measurement process), we can operate and experiment in the symbol field. To some degree this offers an opportunity to anticipate possibilities and/or to predict potential consequences of acting in reality. In this sense, experimenting in symbols precedes acting in reality. We think that economists should always keep in mind the relation which exists between the symbols and their empirical content. Of course, one could reply that, once symbols are derived from the object field the position is legitimated that symbols get a life of their own or, as Debreu puts it, "As a formal model of an economy acquires a mathematical life of its own, it becomes the object of an inexorable process in which rigor, generality, and simplicity are relentlessly pursued" (Debreu, 1986, p. 1265). With these attributes of mathematical process economic theory is perceived as axiomatized:

> An axiomatized theory first selects its primitive concepts and represents each one of them by a mathematical object. For instance the consumption of a consumer, his set of possible consumptions, and his preferences are represented respectively by a point in the commodity space, a subset of the commodity space, and a binary relation in that subset. Next assumptions on the objects representing the primitive concepts are specified, and consequences are mathematically derived from them. The economic interpretation of the theorems so obtained is the last step of the analysis. According to this schema, and axiomatized theory has a mathematical form that is completely separated from its economic content (Debreu, 1986, p. 1265).

We would criticize Debreu's position in so far as we believe that there is a danger of losing contact with the symbols' empirical content while treating the abstract concepts now as a mathematical object. What Debreu *de facto* does is transform the original economic problem into a pure mathematical one and it is the mathematics theory that forms the problems to be analyzed and not the economic theory or any real-life economic activity. With this in mind the conclusion that an "axiomatized theory has a mathematical form that is completely separated from its economic content" has to follow, but only when arguing from the pure mathematical point of view. From the economic viewpoint we have to be sure that the economic (empirical) content, from which we originally started, and which was transformed not only into symbols but also equipped with an "abstract" content (mathematical object), has not changed to a different economic content (see also Dore, 1988, p. 460; Mirowski, 1986, p. 203). Yet there are additional requirements to fullfill during the process of transforming: The meaningfulness of the mathematical operations in the symbol area must guarantee or imply the meaningful transformation back to the object area. Furthermore you have to ensure that you do not ascribe meanings to the symbols in the results which they did not possess when they were introduced. This was pointed out by Dennis (1982). This is especially acute when we are faced with variables of qualitative or psychological magnitude, such as "forcing", "wishing", "must", "belief" (Beed and Kane, 1991, p. 591).

To avoid such problems it is necessary to choose mathematical transformation methods which intend "the breaking down of a complex chain of reasoning into an explicitly stated series of steps, each of which is sufficiently simple for there to be an agreed procedure for dealing with it" (Backhouse, 1998, p. 1849). If we neglect the correspondence between symbol area and economic content we risk playing with mathematical symbols in economics and not applying them for constructive use, as leading economists such as Frisch and Leontief have already pointed out (Frisch, 1933, p. 3; Leontief, 1971, p. 1; 1982, p. 104).

What is the use of mathematics within economics? A conclusion

Mathematic reasoning is one way of logical deduction, which secures a style of logical consistency in reasoning. It is a fallacy to believe that mathematical reasoning and only mathematical reasoning can secure logical consistent reasoning. Mathematical reasoning facilitates the control of consistency and therefore the use of mathematics is useful and helpful, especially in preventing economists from wrong intuitions. Sometimes the use of mathematics even offers undiscovered logical possibilities, which, of course, always can only be a part of the assumptions. But the real importance of the result of analysis can only be judged on the basis of the underlying model, its adequacy of the process of measurement, the adequate correspondence of operations in the real sphere and their formalization by mathematical operators in the symbol field, and the adequacy of the realism of variables, i.e. empirical content, and the conformity of the derived results with empirical data. Mathematization for the sake of mathematization is useless.

The basis of a serious model building is experience. Here we are referred to the hermeneutic background, to aspects of "*Verstehen*", in deciding what we measure and quantify in which way. Mathematical symbolism can certainly not express the wide range of human activities and relationships, expressible in natural language, e.g. dialectical concepts (e.g. Georgescu-Roegen, 1979). We agree with those philosophical positions, reflecting as a profound commonality of reason itself, that "Whenever methods are being employed their correct application is not specified by a method but demands our own judgment . . . All methods require judgment and linguistic instruction" (Gadamer, 1997, p. 367). We do not use the term hermeneutic as a well founded philosophical concept but we use it on a very rudimentary level, that is to say we seek constantly and continuously to assign meaning to our action.

The task of science is to enable us to make better daily decisions. This can be done by formulating working hypotheses better and more adequately. Science (at least applied science) should be a useful help to master life insofar as science serves as a basis for our decisions and these decisions have a substantial impact on reality. Thereby another normative, or say ethical, aspect emerges, a kind of categorical imperative: Do your best to justify your decisions! Do your best to find explanations as a basis to change reality in a helpful direction! Be aware that you are responsible – and assume this responsibility – for all your actions because in a world where truth is not a fruitful concept, no authority can release you from the responsibility for your behavior.

As economists we conclude with an economist:

> There surely is no philosopher's stone in economic method – no one approach whose unqualified success and power mean that it deserves to replace all (or even any) of its rivals (Baumol, 1991, p. 2).

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